

## **MULTI-EQUIPMENT CONDITION BASED MAINTENANCE OPTIMIZATION BY MULTI-OBJECTIVE GENETIC ALGORITHM**

**Štefan Valčuha<sup>1</sup>, Aitor Goti<sup>2</sup>, Juraj Úradníček<sup>3</sup>, Ivan Navarro<sup>4</sup>,**

<sup>1,3</sup>STU Bratislava, Faculty of Mechanical Engineering, Institute of manufacturing system,  
environmental technology and quality management, <sup>2</sup>Natra Group, Ctra. de Aránzazu,

<sup>4</sup>Mondragon Unibertsitatea University, Industrial Management Department,

[stefan.valcuha@stuba.sk](mailto:stefan.valcuha@stuba.sk), [aitor.goti@natra.es](mailto:aitor.goti@natra.es), [juraj.uradnicek@stuba.sk](mailto:juraj.uradnicek@stuba.sk),

[aoyarbide@eps.mondragon.edu](mailto:aoyarbide@eps.mondragon.edu).

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**Abstract:** This paper deals with the optimization of the condition based maintenance (CBM) applied on manufacturing multi-equipment system under cost and benefit criteria. The system is modeled using Discrete Event Simulation (DES) and optimized by means of the application of a Multi-Objective Evolutionary Algorithm (MOEA). The developed approach has been successfully applied to the optimization of condition based maintenance activities of a hubcap production system composed by three plastic injection machines and a painting station, for management decision support.

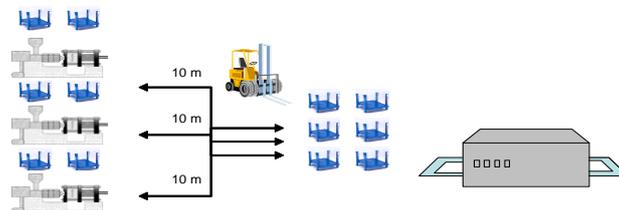
### **1. INTRODUCTION**

This paper provides a solution for the joint optimization of CBM strategies applied on several equipments. Precisely, the research is focused on the problem of CBM optimization in a manufacturing environment with the objective of determining the optimal component deterioration levels or thresholds when preventive maintenance (PM) is performed for multi-equipment systems under cost and profit criteria. The approach developed takes into account the sections interaction of production, work in process material, quality and maintenance aspects. For this purpose, a model that considers maintenance, productive speed loss and non-quality costs along with productive profit has been developed. The model has been implemented using DES and optimized using a MOEA

### **2. OPTIMIZATION PROBLEM**

#### **2.1. SYSTEM DEFINITION**

The system consists of three identical plastic injection machines and a painting station, as it is described in Fig. 1:



**Fig. 1. Configuration of the simplified plastic injection system**

Each machine of the model consists of three subsystems (which are modelled as components) organized in serial configuration, and one maintenance activity is executed over each subsystem in order to control its aging: M1, M2 and M3 are respectively applied over sub-systems S1, S2 and S3 of the injection machines while M4, M5 and M6 are respectively executed on sub-systems S4, S5 and S6 of the painting station. The influence of each subsystem on the performance of each machine is defined in Table 1. for the

injection machine, S1's deterioration influences only unavailability, S2's deterioration affects unavailability and productive speed loss and, S3's deterioration has an effect on unavailability and quality. Similarly, considering the painting station, S4's deterioration influences only unavailability, S5's deterioration affects unavailability and productive speed loss and, S6's deterioration has an effect on unavailability and quality.

**Table 1. System components, PM activities and their influences on productive parameters**

Maintained equipment	Subsystem	PM activities performed	Influences on
Injection machines	M1	S1	Unavailability
	M2	S2	Unavailability and Productive speed loss
	M3	S3	Unavailability and Quality
Painting station	M4	S4	Unavailability
	M5	S5	Unavailability and Productive speed loss
	M6	S6	Unavailability and Quality

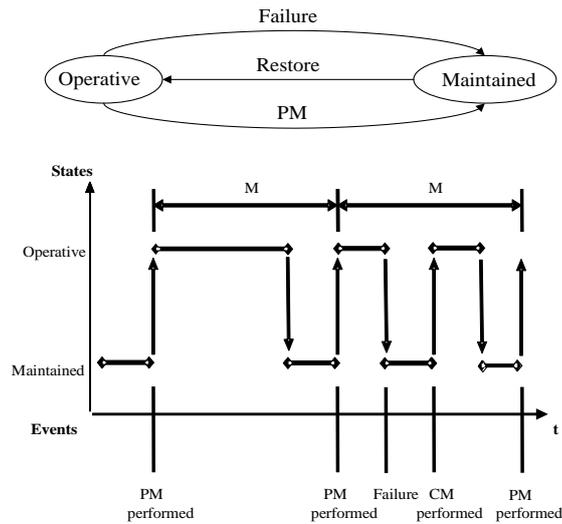
According to obtained statistical data, the equipments failure process is modeled by using a two-parameter ( $\lambda_1, \gamma_1$ ) Weibull failure rate. Additionally, it is considered that the production process can be subject to a process deterioration that shifts the system from an under-control state to an out-of-control state. This process deterioration follows also a Weibull distribution of parameters  $\lambda_2, \gamma_2$ . Table 2. shows the Weibull reliability data for the studied problem.

**Table 2. Weibull data of the studied subsystems**

Group	$\lambda_1(10^{-2}\text{hrs}^{-1})$	$\gamma_1$	$\lambda_2(10^{-2}\text{hrs}^{-1})$	$\gamma_2$
S1	5	2		
S2	2	2.9		
S3	4	2	4	2
S4	6.6	2		
S5	7.7	3		
S6	10	3	10	3

## 2.2. DISCRETE EVENT SIMULATION MODEL

DES concerns the modeling of a system as it evolves over time by a representation in which variable states change suddenly at separate points in time. These changes happened in the system are considered events. Systems do not change between events, so DES considers that it is not necessary to analyze what happens in a system in periods taken place between two events. Fig. 2 shows an example of how DES generates events in a machine that may be operative or stopped due to CM or PM.



**Fig. 2. States and events of a simplified machine**

The DES model simulates the injection machines, the painting station, the lift, the product buffers and its pallets. The implementation of each of these components is detailed in the following subsections.

### Equipment modeling

That equipment model was developed considering the following assumptions: 1) the effect of the maintenance activities is modeled by using an imperfect maintenance model. In this case a Proportional Age Set-Back (PAS) [1] is assumed, 2) the failure process and deterioration process are independent, 3) the system only produces non-conforming items, with a rate constant ( $\alpha$ ), while the process is out-of-control, 4) Preventive maintenance and process inspection are performed simultaneously, 5) inspections are error free and 6) the process is restored to under control state when the preventive maintenance is realized, 7) productive speed is assumed to fall from its initial speed ( $V_0$ ) to another speed value ( $V^*(x)$ ) which depends on the CBM deterioration threshold, 8) as in [2], we assume that all the deterioration processes of the three studied components are independent, and 9) it is assumed that the process produces a single product type, so setup times of reference changes are not simulated.

Considering a CMT strategy, PM is performed when the component gets a determined critical age or deterioration level ( $w_c$ ). It is worth remembering that PAS model considers that the maintenance reduces proportionally, in a  $\varepsilon$  factor, the age that the component has immediately before it enters maintenance ( $w_m^-$ ). Considering these conditions, maintenance always will be applied to a component when it has the same age, and as effectiveness is assumed to be constant, the age of the component will always be the same after performing a PM action ( $w_m^+$ ). This means that  $w_m^-$  and  $w_m^+$ , which represent respectively the age of the component just before and after the  $m$ -th PM intervention, will always get the same values:

$$w_m^- = w_c \tag{1}$$

$$w_m^+ = (1 - \varepsilon) \cdot w_c \tag{2}$$

As a consequence, the time interval  $M$  between two PM activities will also be equal to a constant:

$$M = w_c \cdot \varepsilon \quad (3)$$

The relevant productive parameters of the described equipment model include: i) direct maintenance parameters, ii) quality parameters and iii) productive speed loss parameters. These parameters can be evaluated as:

$$u_{pm}(\mathbf{x}) = \frac{d_{pm}}{M} \quad (4)$$

$$u_{cm}(\mathbf{x}) = \left[ \rho + (1 - \rho) \cdot \left( 1 - e^{-h^*(\mathbf{x}) \cdot M} \right) \right] \cdot \frac{d_{cm}}{M} \quad (5)$$

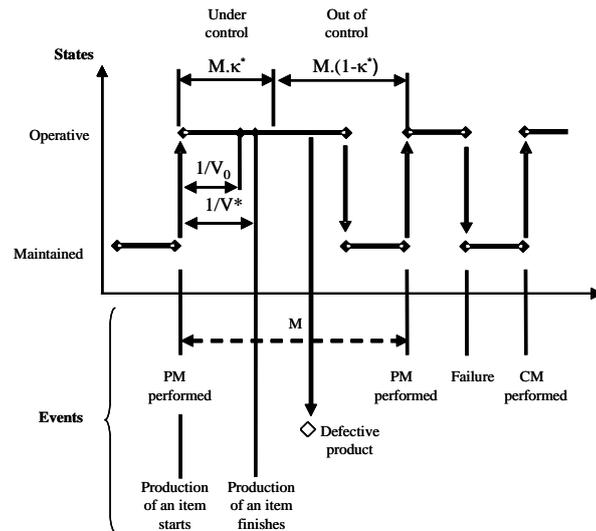
$$V^*(\mathbf{x}) = V_0 - \left[ \tau \cdot M \cdot \left( \frac{2 - \varepsilon}{2\varepsilon} \right) \right] \quad (6)$$

$$\kappa^*(\mathbf{x}) = \frac{1}{M} \int_0^M t \cdot f_m(w(t, \varepsilon)) dw \approx \frac{1}{2} \cdot h^*(\mathbf{x}) \cdot e^{-h^*(\mathbf{x}) \cdot M} \quad (7)$$

where  $u_{pm}(\mathbf{x})$  represents the unavailability contribution of a component as a consequence of the PM activities performed on the component in the analysis period  $L$ ;  $u_{cm}(\mathbf{x})$  the unavailability contribution of a component as consequence of performing CM;  $V^*(\mathbf{x})$  the mean production speed of the equipment during the  $L$  period; and  $\kappa^*(\mathbf{x})$  the mean fraction of time where the process is under control. In addition, the following notation is used:  $M$  is the PM intervals performed when the deterioration level of the equipment achieves a certain value;  $d_{pm}$  the mean time for PM;  $d_{cm}$  the mean time for CM;  $V_0$  the initial (e.g. as per design) production speed;  $\tau$  the speed loss coefficient;  $\rho$  the cyclic or per-demand failure probability;  $h^*(\mathbf{x})$  the averaged failure rate;  $\varepsilon$  the effectiveness of the PM activity; and  $f_m(w(t, \varepsilon))$  the density function obtained using the conditional hazard function.

In this research, analytical formulation corresponding to each machine of the productive system is implemented within the equipment to generate stochastic events that make equipment work as it is defined in the analytical model. This integration is performed in two steps: first the components of the decision vector related to the studied machines are evaluated analytically, obtaining the working parameters  $U_{cm}(\mathbf{x})$ ,  $U_{pm}(\mathbf{x})$ ,  $V^*(\mathbf{x})$  and  $\kappa^*(\mathbf{x})$  of the corresponding PM frequencies (where  $U_{cm}(\mathbf{x})$  and  $U_{pm}(\mathbf{x})$  are respectively the unavailability of a machine due to corrective maintenance (CM) and PM, evaluated using the system fault tree and the single component  $u_{cm}(\mathbf{x})$  and  $u_{pm}(\mathbf{x})$  contributions). In a second step, the generated working parameters are introduced as inputs in the DES modelled machines to execute then a simulation where the results to be optimised are obtained.

The implementation of values obtained in the analytical evaluation executed in the DES model derives in the generation of planned PM, unplanned CM, speed reduction and defective product actions and events during the simulation. As a consequence, at the end of the simulation machines generate the same values of  $U_{cm}(\mathbf{x})$ ,  $U_{pm}(\mathbf{x})$  and  $\kappa^*(\mathbf{x})$  defined by the analytical model to produce items in a  $V^*(\mathbf{x})$  productive speed. Fig. 3 shows the generation of unavailability, speed loss and quality events for an equipment during a simulation:



**Fig. 3. Generation of events related to maintenance, productive speed and quality**

As it can be seen in Fig. 3 events related to PM are generated with a determined periodicity ( $M$ ) and each product needs a  $1/V^*$  cycle time to be produced. Failures are generated randomly to obtain an unavailability related to CM which is equal to  $U_{cm}(\mathbf{x})$ . Referred to quality, there are no defective products during the first  $\kappa^*(\mathbf{x})$  fraction between two PM activities, while there is a  $\alpha$  defective fraction during the following  $(1-\kappa^*(\mathbf{x}))$  fraction. Thus, thanks to the interaction between analytical evaluation and DES modelling simulation equipments work as it is defined in analytical models shown in Eqns. (4 – 7). Additionally, and thanks to the capability of combining different machines in a system, the DES model not only models the features of a single machine, but the interaction among several machines.

The generation of each one of the above mentioned events is related to a specific inefficiency so their costs have to be taken into account. Costs are quantified considering CM, PM, speed loss and quality terms. In order to do that, individual cost counters related each one of these terms ( $c_{cm}(\mathbf{x})$ ,  $c_{pm}(\mathbf{x})$ ,  $c_{sl}(\mathbf{x})$  and  $c_q(\mathbf{x})$ , respectively) are defined; these counters are initialized to zero at the beginning of the simulation and increased every time an event related to them is generated by the simulation using Eqns. (8 – 11):

$$c_{cm}(\mathbf{x}) = c_{cm}(\mathbf{x}) + d_{cm} \cdot c_{hcm} \quad (8)$$

$$c_{pm}(\mathbf{x}) = c_{pm}(\mathbf{x}) + d_{pm} \cdot c_{hpm} \quad (9)$$

$$c_{sl}(\mathbf{x}) = c_{sl}(\mathbf{x}) + \left[ \left( \frac{1}{V^*(\mathbf{x})} \right) - \left( \frac{1}{V_0} \right) \right] \cdot c_{hsl} \quad (10)$$

$$c_q(\mathbf{x}) = c_q(\mathbf{x}) + c_\alpha \quad (11)$$

where  $c_{hcm}$ ,  $c_{hpm}$  and  $c_{hsl}$  represent respectively the hourly cost related to the CM, the PM and the reduced speed, while  $c_\alpha$  represents the cost of manufacturing a defective product. Finally,  $P(\mathbf{x})$  characterizes the profit function obtained as a result of selling non-defective products, which can be evaluated as:

$$P(\mathbf{x}) = n(\mathbf{x}) \cdot \psi \quad (12)$$

where  $n(\mathbf{x})$  represents the amount of non-defective products obtained during the analysis period ( $L$ ), and  $\psi$  is the estimated margin of a single product.

### Buffer and transportation modeling

System buffers have a determined maximum capacity. The model assumes that if a buffer is full it will not receive any products until it has free pallets to store them (so the transportation events will not be executed).

Referred to transportation modeling, only semi-elaborated product movements have been modeled, considering movements between: i) a machine and a buffer location, ii) two machines, iii) a buffer location and a machine, and iv) two buffer locations. It is worth to note that for transportation types i), ii) and iii) products are moved one by one, whereas for movements between two buffer locations products are transported in pallets.

### Simulation values of the productive system

Data collected for the simulation model is shown in the next 4 tables. Tables 3. and 4. show parameters related to PM and CM, whereas Tables 5. and 6. detail respectively information about inputs related to CM, unavailability, speed, quality and cost for the injection machines and the painting station.

**Table 3. PM data related to the productive system**

Activity maintenance	$\varepsilon$	$d_{pm}$ (hrs)
M1	0.9	0.5
M2	0.9	1
M3	0.9	1
M4	0.9	2
M5	0.9	1
M6	0.9	3

**Table 4. CM data related to the productive system**

Corrective breakdown of sub-system	$d_{cm}$ (hrs)
S1	0.5
S2	1
S3	2
S4	0.5
S5	1
S6	2

**Table 5. Productive and cost parameters for the injection machines**

$C_a(\text{€}/u^1)$	6
$\tau (u/h^2)$	0.0017
$C_{hsl}(\text{€}/hr)$	25
$\rho(10^{-3})$	1
$\alpha$	0.03
$h_0(\text{fail}/hr)$	0
$V_0(u/hr)$	180
$c_{hcm}(\text{€}/hr)$	45
$c_{hpm}(\text{€}/hr)$	30

**Table 6. Productive and cost parameters for the painting station**

$C_a(\text{€}/\text{u}^1)$	6
$\tau (\text{u}/\text{h}^2)$	0.02
$C_{\text{hsi}}(\text{€}/\text{hr})$	150
$\rho(10^{-3})$	1
$\alpha$	0.04
$h_0(\text{fail}/\text{hr})$	0
$V_0(\text{u}/\text{hr})$	900
$c_{\text{hcm}}(\text{€}/\text{hr})$	175
$c_{\text{hpm}}(\text{€}/\text{hr})$	160
$c_{\text{hcbm}}(\text{€}/\text{hr})$	2

Additionally, the net profit value of a non-defective product ( $\psi$ ) is 0.2 €/unit and the simulation time  $L$  is 62400 working hours, which corresponds to 10 years of production working 5 days a week and 24 hours a day.

## 2.2. THE NSGA-II MULTI-OBJECTIVE EVOLUTIONARY ALGORITHM

In this approach the Non-dominated Sorting Genetic Algorithm (NSGA-II) proposed by Deb et al. [3] has been implemented. The NSGA-II is the most recent and improved version of the NSGA which incorporates: a) a faster non-dominated sorting approach, b) an elitist strategy i.e. the best non-dominated individuals are preserved from one generation to another by using a crowding measurement, and c) no niching parameter.

- Step 1.* Fix  $N$ ,  $i=1$ , and  $i_{\text{max}}$ .
- $N$  = population size
  - $i$  = number of generations
  - $i_{\text{max}}$  = maximum number of interactions of the genetic algorithm
- Step 2.* Create and evaluate a random parent population  $P_i$  of size  $N$ .
- Step 3.* If  $i=i_{\text{maxGA}}$  return  $P_i$  else:
- Step 4.* Form a combined population of size  $2N$  as  $T_i = P_i \cup Q_i$ .
- $Q_i$  = offspring population
  - $T_i$  size  $N$  and equal to  $P_i$  in the first interaction
- Step 5.* Ranking (according to restriction violations).
- Step 6.* Identify non dominated fronts  $F_1, F_2, \dots, F_k$ . Thus an each solution is assigned a fitness equal to its non-domination level.
- Step 7.* Create  $P_{i+1}$  as the  $N$  best individuals from  $P_i$ .
- Step 8.* Select randomly  $N$  couples from  $P_{i+1}$  using a binary tournament selection.
- Step 9.* Create offspring population  $Q_{i+1}$  applying crossover and mutation (size  $N$ ).
- Step 10.* Evaluate the offspring population.
- Step 11.* Do  $i=i+1$ .
- Step 12.* Go to step 4.

Following the procedure detailed above the algorithm evaluates the  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N$  genes of each generation. In this case, to obtain the respective  $f(\mathbf{x}_1), f(\mathbf{x}_2), \dots, f(\mathbf{x}_N)$  fitness values of the evaluation, the DES model performs a simulation where deterioration limit levels (observed through CBM) where PM activities are launched act as decision variables to obtain economic parameters.

## 2.3. PROBLEM FORMULATION

The optimization of preventive maintenance activities based on cost and benefit criteria can be formulated as a multi-objective optimization problem (MOP). A general MOP includes a set of parameters (decision variables), a set of objective functions, and a set of constraints. Objective functions and constraints are defined in terms of the decision

variables using the models presented in the previous section. The optimization goal can be formulated to optimize a vector of functions of the form [4]:

$$\mathbf{y} = f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \dots, f_n(\mathbf{x})) \quad (13)$$

subject to the vector of constraints

$$\mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}), g_2(\mathbf{x}), \dots, g_n(\mathbf{x})) \quad (14)$$

where

$$\mathbf{x} = \{x_1, x_2, \dots, x_n\} \in \mathbf{X} \quad (15)$$

$$\mathbf{y} = \{y_1, y_2, \dots, y_n\} \in \mathbf{Y} \quad (16)$$

and  $\mathbf{x}$  is the decision vector (vector of decision variables),  $\mathbf{y}$  the objective vector,  $\mathbf{X}$  the decision space and  $\mathbf{Y}$  is the objective space, that is to say  $\mathbf{Y}=f(\mathbf{X})$ .

The optimization of deterioration levels when PM activities are launched proposed in this paper considers the productive costs and profit as optimization criteria. Both cost and profit models depend on the decision vector,  $\mathbf{x}$ . So, the vector of bi-objective function,  $f(\mathbf{x})$ , is defined as:

$$f(\mathbf{x}) = \{C(\mathbf{x}), P(\mathbf{x})\} \quad (17)$$

where the objective is to minimize the function  $C(\mathbf{x})$  and maximize a profit function  $P(\mathbf{x})$ .  $C(\mathbf{x})$  is the cost system which is evaluated as sum of the maintenance, production speed lost and quality costs for each of the  $m$  machines of the system which are evaluated using Eqns. (8 – 11).

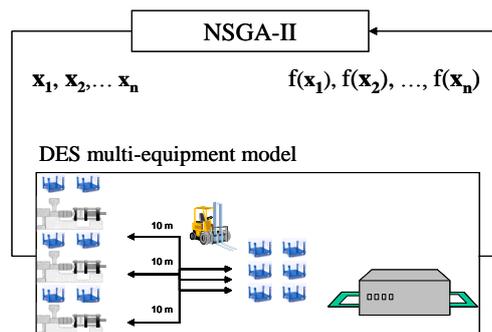
$$C(\mathbf{x}) = \sum_{i=1}^m (c_{cm_i}(\mathbf{x}) + c_{pm_i}(\mathbf{x}) + c_{sl_i}(\mathbf{x}) + c_{q_i}(\mathbf{x})) \quad (18)$$

and  $P(\mathbf{x})$  is the profit function obtained as a result of selling non-defective products, evaluated as it is detailed in Eq. (12).

In this case there are no constraints defined in terms of the vector of constraints. Nevertheless, constraints are imposed directly over the values the decision variables can take, which must get typified values, representing each one a day, two days, etc.

This maintenance optimization MOP can be solved using a MOEA. A MOEA is a multi-objective search method based on Darwin's evolutionary theory applied to a population of possible solutions which evolves and tends to converge to an optimal solution set.

The MOEA, in this case the NSGA-II, evolves the population which is evaluated executing simulations by using the developed model. The scheme of the optimization approach is shown in Fig. 5:

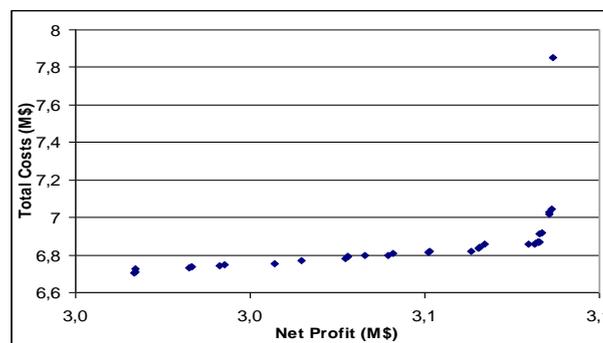


**Fig. 4. Optimization approach**

As it can be seen in Fig. 4, the NSGA-II creates a population of  $n$  decision vectors ( $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n$ ) which are evaluated executing simulations. The model returns the fitness values of each one of these vectors ( $f(\mathbf{x}_1), f(\mathbf{x}_2), \dots, f(\mathbf{x}_n)$ ) which are processed in the NSGA-II to generate new populations. These evolutions tend to achieve solutions which are located in a Pareto optimal front, where it cannot be determined that a solution obtained is better than another without considering additional information.

## 2.4. RESULTS

Fig. 5 represents a cost plot of results found by the NSGA-II. The results shown were calculated using a Pentium 4 3.2 GHz 1 GB RAM running the MOEA evolving a population of size 50 individuals for 200 generations with a selection rate of 0.25, crossover rate of 0.5 and mutation rate of 0.75. The DES model was using Witness 2008® while the NSGA-II was implemented in Matlab R2009



**Fig. 5. Pareto front obtained in the optimization process**

## 3. CONCLUSIONS

Solution for the joint optimization of the condition based maintenance model applied on several equipment has been obtained. Developed approach takes into account the section interaction of production, work in process material, quality and maintenance aspects. Model has been implemented using discrete event simulation (DES) and optimized using Multiobjective Evolutionary Algorithm (MOEA). Pareto frontier of the multi objective optimization process has been obtained.

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